Two-Phase Laminar Wall Jet Flow With Electrification of Particles

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Abstract — *A laminar two-phase incompressible boundary layer flow through a wall jet has been investigated with electrification and Brownian diffusion of particles. The particles are charged even after in the absence of external electric field due to particle-particle and particle-wall interaction. Schlichting perturbed model is applied to find the momentum boundary layer solution of mean as well as perturbed flow. The solution of momentum mean fluid flow is obtained analytically. The other flow profiles are solved by using finite difference technique with non-uniform grid to analyze in terms of graphs.*

Keywords—*Two-phase flow, Momentum boundary layer, electrification of particles, Wall Jet, Brownian diffusion, Non-uniform grid.*

I. Introduction

Gas-particle flows, dusty fluid flows and the flow of suspensions have received considerable attention due to the importance in various engineering applications. The influence of suspended particles in both natural and industrial processes like sand dust storms, tornados, volcano eruptions, fluidized beds, coal classifiers, power conveyers, particle-laden jets, petroleum industry, purification of crude oil, manufacturing in the chemical, pharmaceutical, biomedical, mineral and new materials sectors, and increasingly grow in importance as new techniques and applications, such as functional nano materials are developed. One important engineering application is the predication and prevention of dust fires and explosions in plants, storerooms and coal mines. It is well known that many organic or metallic powders like cornstarch, coal, aluminium and magnesium are suspended in air form explosive mixtures due to huge specific surface area of fine dispersed particles. Dust fires or explosions may occur if the ignition-source initiation energy is very high. Usually, explosive particles are deposited down or piled up on the floor and it is quite difficult to ignite them. However, the particles may be aerodynamically entrained into the air flow, which can be induced by a primary gaseous explosion, propagates over the deposit layer. This leads in the formation of dust cloud and increase the air temperature. In addition to these applications, dust particles in the boundary layer includes soil erosion by natural winds and dust entrainment in a cloud during a nuclear explosion flow in rocket tubes, combustion, paint spraying and blood flow in capillaries. In all these applications, a basic understanding of how particles interact with the fluid flows is necessary to allow the use of computational fluid dynamics (CFD) models in the optimization and performance improvement of existing equipment and processes, the identification and solution of operating problems, the evolution of retrofit options, and the design of new equipment systems and plant, including process scale-up.

Many investigators have developed and analysed the multiphase flow equations. Soo [16] had developed the mathematical approach to this type of flows. A detailed derivation of the momentum equations for disperse two-phase systems was studied by Rietema and van der Akker [8]. Osiptsov[1] reviewed the mathematical modelling of dusty-gas laminar boundary layers in the framework of the two-phase fluid approach. He formulated the two phase boundary layer approximation, used the matched asymptotic expansion method and studied accumulation of the particles in the boundary layers, effects of particles on the wall shear stress and heat fluxes. Saffman [10] investigated the stability of the laminar flow of a dusty gas in which the dust particles are uniformly distributed. Palani and Ganesan [3] have used the implicit finite difference method to study the velocity and heat transfer aspects on the dusty-gas flow past on a semi-infinite isothermal inclined plate. Most of the authors [11-14] have studied two phase boundary layer profiles along with the skin friction and heat transfer and the authors [2,5-7, 17-19] are investigated the jet flows for clear fluid as well as fluid with particles. But none of the authors have investigated the flow field with electrification of particles through a wall jet.

In the Present analysis, the particles will be allowed to diffuse through the carrier fluid i.e. the random motion of the particles shall be taken into account because of the small size of the particles. Particle cloud is treated as a fluid and to calculate the particle concentration in terms of diffusion equation in place of particle continuity equation. The momentum boundary layer characteristics have been studied by employing finite difference technique using non-uniform grid.

Although electric charge on the solid particles can be excluded by the definition in theoretical analysis or when dealing truly with a boundless system, electrification of the solid particles always occurs when contact and separation are made between the solid particles and a wall of different materials or similar materials but different surface condition. The electric charges on the solid particles cause deposition of the solid particles on a wall in a more significant manner than the gravity effect and are expected to affect the motion of a metalized propellant and its product of reaction through a rocket nozzle and the jet at the exit of the nozzle. The charged solid particles in the jet of a hot gas also effect radio communications. Therefore, we have considered the effect of electrification of particles on the flow of a plane wall jet

II. MATHEMATICAL MODELLING

Let an incompressible fluid with suspended particulate matter(SPM) be discharged through a narrow slit in the half space along a plane wall and mixed with the same surrounding fluid being initially at rest having temperature T_∞ . The wall is also maintained at the same constant temperature T_∞.Taking the origin in the slit and the co-ordinate axis x and y along and normal to the plane wall respectively.

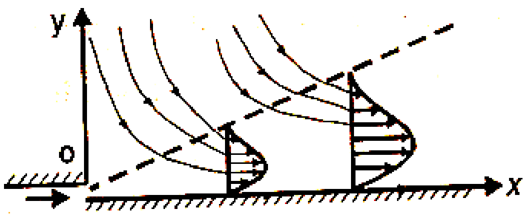


Fig.1 : Plane wall jet

Neither the particles are charged nor any electric/ magnetic field is supplied to the flow field. But the particles are charged due to particle-particle and wall –particle interaction. So, the terms due to electrification of particles are considered in the momentum equations for both the phases, which may not be neglected. Under the above assumptions, the boundary layer equations for the flow field are given by

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$\frac{\partial}{\partial x}(\rho_p u_p)+\frac{\partial}{\partial y}(\rho_p v_p)=0 \tag{2}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}-\frac{1}{1-\varphi}\frac{1}{\tau_p}\frac{\rho_p}{\rho}(u-u_p)+\frac{1}{1-\varphi}\frac{\rho_p}{\rho}\left(\frac{e}{m}\right)E \tag{3}$$

$$u_p\frac{\partial u_p}{\partial x}+v_p\frac{\partial u_p}{\partial y}=\nu_s\frac{\partial^2 u_p}{\partial y^2}+\frac{1}{\tau_p}(u-u_p)+\left(\frac{e}{m}\right)E \tag{4}$$

and the diffusion equation

$$u_p\frac{\partial \rho_p}{\partial x}+v_p\frac{\partial \rho_p}{\partial y}=\nu_p\frac{\partial^2 \rho_p}{\partial y^2} \tag{5}$$

Introducing the non-dimensional quantities like

$$x^*=\frac{x}{L},\ y^*=\frac{y}{L}\sqrt{Re},\ u^*=\frac{u}{U},\ v^*=\frac{v}{U}\sqrt{Re},\ u_p^*=\frac{u_p}{U},\ v_p^*=\frac{v_p}{U}\sqrt{Re},\ \rho_p^*=\frac{\rho_p}{\rho_{p_0}} \tag{6}$$

in the above boundary layer equations (1) to (5), and after dropping stars,

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{7}$$

$$\frac{\partial}{\partial x}(\rho_p u_p)+\frac{\partial}{\partial y}(\rho_p v_p)=0 \tag{8}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{\partial^2 u}{\partial y^2}-\alpha\frac{1}{1-\varphi}\frac{FL}{U}\rho_p(u-u_p)+\frac{1}{1-\varphi}\alpha M\rho_p \tag{9}$$

$$u_p\frac{\partial u_p}{\partial x}+v_p\frac{\partial u_p}{\partial y}=\epsilon\frac{\partial^2 u_p}{\partial y^2}+\frac{FL}{U}(u-u_p)+M \tag{10}$$

$$u_p\frac{\partial \rho_p}{\partial x}+v_p\frac{\partial \rho_p}{\partial y}=\epsilon\frac{\partial^2 \rho_p}{\partial y^2} \tag{11}$$

Subjected to the boundary conditions

$$y=0:\ u=0,\quad u_p=u_{p_w},\ \rho_p=\rho_{p_w} \tag{12a}$$

$$y=\infty:\ u=u_p=0,\ \rho_p=0, \tag{12b}$$

and the integral conditions

$$\frac{\partial}{\partial x}\int_0^\infty\left\{u^2\left(\int_0^y u\,dy\right)\right\}dy+\frac{1}{1-\varphi}\frac{FL}{U}\alpha\int_0^\infty\left\{u\int_y^\infty\rho_p(u-u_p)dy\right\}dy-\frac{1}{1-\varphi}\alpha M\int_0^\infty u\left(\int_y^\infty\rho_p dy\right)dy=0 \tag{13}$$

III. METHOD OF SOLUTION

The above non-dimensional equations are solved by Schlichting [4] perturbation method. The boundary layer profiles are expressed in terms of mean flow and perturbed flow as

$$u=u_0+u_1,\ u_p=u_{p_0}+u_{p_1},\ \rho_p=\rho_{p_0}+\rho_{p_1} \tag{14}$$

where '$_0$' represents the mean flow and '$_1$' for perturbed flow.

Using the above Schlichting method in equations (7) to (11), we get two sets of equations for mean and perturbed flow as follows.

Mean flow and its solution:

The derived equations for mean flow as

$$\frac{\partial u_0}{\partial x}+\frac{\partial v_0}{\partial y}=0 \tag{15}$$

$$\frac{\partial}{\partial x}\left(\rho_{p_0}u_{p_0}\right)+\frac{\partial}{\partial y}\left(\rho_{p_0}v_{p_0}\right)=0 \tag{16}$$

$$u_0\frac{\partial u_0}{\partial x}+v_0\frac{\partial u_0}{\partial y}=\frac{\partial^2 u_0}{\partial y^2}-\frac{1}{1-\varphi}\frac{FL}{U}\alpha\,\rho_{p_0}\left(u_0-u_{p_0}\right)+\frac{1}{1-\varphi}\alpha M\rho_{p_0} \tag{17}$$

$$u_{p_0}\frac{\partial u_{p_0}}{\partial x}+v_{p_0}\frac{\partial u_{p_0}}{\partial y}=\epsilon\frac{\partial^2 u_{p_0}}{\partial y^2}+\frac{FL}{U}\left(u_0-u_{p_0}\right)+M \tag{18}$$

$$u_{p_0}\frac{\partial \rho_{p_0}}{\partial x}+v_{p_0}\frac{\partial \rho_{p_0}}{\partial y}=\epsilon\frac{\partial^2 \rho_{p_0}}{\partial y^2} \tag{19}$$

Subjected to the boundary conditions

$$y=0:\ u_0=0,\quad u_{p_0}=u_{p_{wo}},\ \rho_{p_0}=\rho_{p_{wo}} \tag{20a}$$

$$y=\infty:\ u_0=u_{p_0}=0,\ \rho_{p_0}=0 \tag{20b}$$

with the integral condition

$$\frac{\partial}{\partial x}\int_0^\infty\left\{u_0^2\left(\int_0^y(u_0)dy\right)\right\}dy+\frac{1}{1-\varphi}\frac{FL}{U}\alpha\int_0^\infty\left\{u_0\int_y^\infty\rho_{p_0}\left(u_0-u_{p_0}\right)dy\right\}dy-\frac{1}{1-\varphi}\alpha M\int_0^\infty u_0\left(\int_y^\infty\rho_{p_0}dy\right)dy=0 \tag{21}$$

Since we are considering the case of a dilute suspension of particles, following Soo[20], the velocity distribution in the fluid is not significantly affected by the presence of the particles. Therefore the drag force term and the term due to electrification of particles. In the fluid momentum equation is dropped. But for the submicron particles, Brownian motion can be significant, so the concentration distribution equation (16) above will be modified by Brownian diffusion equation (19). With the above consideration the equations (17) and (21) become

$$u_0\frac{\partial u_0}{\partial x}+v_0\frac{\partial u_0}{\partial y}=\frac{\partial^2 u_0}{\partial y^2} \tag{22}$$

$$\frac{\partial}{\partial x}\int_0^\infty\left\{u_0^2\left(\int_0^y(u_0)dy\right)\right\}dy=0$$

$$\text{Or,}\ \int_0^\infty\left\{u_0^2\left(\int_0^y(u_0)dy\right)\right\}dy=A(say) \tag{23}$$

A similar solution of the equation (22) under the present boundary and integral conditions is possible by assuming,

$$\Psi=(Ax)^{1/4}f(\eta),\eta=\left(\frac{A}{1}\right)^{1/4}yx^{-\frac{3}{4}} \tag{24}$$

$$and\ u_0=\frac{\partial\Psi}{\partial y}=\left(\frac{A}{x}\right)^{\frac{1}{2}}f'(\eta),\ v_0=-\frac{\partial\Psi}{\partial x}=\frac{1}{4}\left(\frac{A}{x^3}\right)^{\frac{1}{4}} \tag{25}$$

Where prime denotes differentiation w.r.t. ‘η’.

By considering the above assumption, the equation of continuity is satisfied identically and the equation (22) reduces to,

$$4f'''+ff''+2f'^2=0 \tag{26}$$

With the boundary conditions

$$\eta=0;f=0,f'=0 \tag{27}$$

$$\eta=\infty:f'=o \tag{28}$$

and integral condition

$$\int_0^\infty ff'^2\,d\eta=1 \tag{29}$$

Multiplying by f (Integrating factor) throughout and integrating the equation

$$4ff''-2f'^2+f^2f'=0 \tag{30}$$

Where the constant of integration is zero by using boundary condition.

The differential equation (30) can be linearized if we substitute $f'=\phi$, and considering the function f as the independent variable, we get $f''=\phi\frac{d\phi}{df}$ and the linearized form of the equation (30) is

$$\frac{d\phi}{df}-\frac{1}{2f}\phi=-\frac{f}{4},\ \text{as}\phi\neq 0 \tag{31}$$

Solving above we get

$$\phi=f'=C\sqrt{f}-\frac{1}{6}f^2 \tag{32}$$

Where C is arbitrary constant to be determined.

Assuming at $\eta=\infty, f=f_{\infty,}$ then in view of boundary condition (27 & 28) we get

$$C=\frac{1}{6}f_\infty^{\frac{3}{2}} \tag{33}$$

The value of f_∞ is yet to be determined and for this we use the integral condition (27 & 28) which may be written as

$$\int_0^{f_{\infty,}}ff'df=1 \tag{34}$$

From (34) , we get $\int_0^{f_{\infty,}}f\left(C\sqrt{f}-\frac{f^2}{6}\right)df=1$

$$\text{Or,}\ f_\infty=40^{\frac{1}{4}}=2.515 \tag{35}$$

Now to solve the differential equation (31) we substitute $F=\frac{f}{f_{\infty,}}$ (36)

So that it becomes $\frac{dF}{d\eta}=\frac{f_{\infty,}}{6}\left(\sqrt{F}-F^2\right)$

Solving we get

$$\eta=\frac{2}{f_{\infty,}}\left(ln\frac{1+\sqrt{F}+F}{\left(1-\sqrt{F}\right)^2}+2\sqrt{3}arctg\,\frac{\sqrt{3F}}{2+\sqrt{F}}\right) \tag{37}$$

The solution of particles phase equation are obtained by developing a computational algorithm with non-uniform-grid, Mishra & Tripathy[14], finite difference expressions are introduced for the various terms in equations (18) and (16/19) as,

$$\frac{\partial W}{\partial x}=\frac{1.5\,W_j^{n+1}-2W_j^n+0.5\,W_j^{n-1}}{\Delta x}+o(\Delta x^2) \tag{38}$$

$$\frac{\partial W}{\partial y}=\frac{W_{j+1}^{n+1}-(1-r_y^2)W_j^{n+1}-r_y^2W_{j-1}^{n+1}}{r_y(r_y+1)\Delta y}+o(\Delta y^2) \tag{39}$$

$$\frac{\partial^2 W}{\partial y^2}=2\frac{W_{j+1}^{n+1}-(1+r_y)W_j^{n+1}+r_yW_{j-1}^{n+1}}{r_y(r_y+1)\Delta y^2}+o(\Delta y^2) \tag{40}$$

$$W_j^{n+1}=2W_j^n-W_j^{n-1} \tag{41}$$

$$\text{and}\quad y_{j+1}-y_j=r_y\left(y_j-y_{j-1}\right)=r_y\Delta y_j \tag{42}$$

Where W stands for either u_{p_0} or ρ_{p_0}.

Now the equations (18) and (16/19) reduced to

$$a_j^*W_{o_{j-1}}^{n+1}+b_j^*W_{o_j}^{n+1}+c_j^*W_{o_{j+1}}^{n+1}=d_j^* \tag{43}$$

where W_o stands the mean flow.

Perturbed flow and its solution

The derived equations for perturbed flow as

$$\frac{\partial u_1}{\partial x}+\frac{\partial v_1}{\partial y}=0 \tag{44}$$

$$u_{p_1}\frac{\partial \rho_{p_0}}{\partial x}+\rho_{p_0}\frac{\partial u_{p_1}}{\partial x}+u_{p_0}\frac{\partial \rho_{p_1}}{\partial x}+\rho_{p_1}\frac{\partial u_{p_0}}{\partial x}$$
$$+v_{p_1}\frac{\partial \rho_{p_0}}{\partial y}+\rho_{p_0}\frac{\partial v_{p_1}}{\partial y}+v_{p_0}\frac{\partial \rho_{p_1}}{\partial y}$$
$$+\rho_{p_1}\frac{\partial v_{p_0}}{\partial y}=0 \tag{45}$$

$$u_0\frac{\partial u_1}{\partial x}+u_1\frac{\partial u_0}{\partial x}+v_0\frac{\partial u_1}{\partial y}+v_1\frac{\partial u_0}{\partial y}=\frac{\partial^2 u_1}{\partial y^2}$$
$$-\frac{1}{1-\varphi}\frac{FL}{U}\alpha\,\rho_{p_1}(u_0-u_{p_0})$$
$$-\frac{1}{1-\varphi}\frac{FL}{U}\alpha\,\rho_{p_0}(u_1-u_{p_1})$$
$$+\frac{1}{1-\varphi}\alpha M\rho_{p_1} \tag{46}$$

$$u_{p_0}\frac{\partial u_{p_1}}{\partial x}+u_{p_1}\frac{\partial u_{p_0}}{\partial x}+v_{p_0}\frac{\partial u_{p_1}}{\partial y}+v_{p_1}\frac{\partial u_{p_0}}{\partial y}$$
$$=\epsilon\frac{\partial^2 u_{p_1}}{\partial y^2}+\frac{FL}{U}(u_1-u_{p_1}) \tag{47}$$

$$u_{p_0}\frac{\partial \rho_{p_1}}{\partial x}+u_{p_1}\frac{\partial \rho_{p_0}}{\partial x}+v_{p_0}\frac{\partial \rho_{p_1}}{\partial y}$$
$$+v_{p_1}\frac{\partial \rho_{p_0}}{\partial y}=\epsilon\frac{\partial^2 \rho_{p_1}}{\partial y^2} \tag{48}$$

Subjected to the boundary condition

$$y=0:\ u_1=0,\quad u_{p_1}=u_{p_{w1}},\ \rho_{p_0}=\rho_{p_{w1}} \tag{49a}$$
$$y=\infty:\ u_1=u_{p_1}=0,\ \rho_{p_1}=0 \tag{49b}$$

and the integral condition

$$\frac{\partial}{\partial x}\int_0^\infty\left\{u_0^2\left(\int_0^y(u_1)dy\right)\right\}dy$$
$$+\frac{\partial}{\partial x}\int_0^\infty\left\{2u_0u_1\left(\int_0^y(u_1)dy\right)\right\}dy$$
$$+\frac{1}{1-\varphi}\frac{FL}{U}\alpha\begin{bmatrix}\int_0^\infty\left\{u_0\int_y^\infty\rho_{p_0}(u_1-u_{p_1})dy\right\}dy\\+\int_0^\infty\left\{u_0\int_y^\infty\rho_{p_1}(u_0-u_{p_0})dy\right\}dy\\+\int_0^\infty\left\{u_1\int_y^\infty\rho_{p_0}(u_0-u_{p_0})dy\right\}dy\end{bmatrix}$$
$$-\frac{1}{1-\varphi}\alpha M\int_0^\infty u_1\left(\int_y^\infty\rho_{p_0}dy\right)dy$$
$$-\frac{1}{1-\varphi}\alpha M\int_0^\infty u_0\left(\int_y^\infty\rho_{p_1}dy\right)dy=0 \tag{50}$$

is identically satisfied.

Using equations (38) to (42) in (44) and (45) to (48), we get

$$v_{1j}^{n+1}=v_{1j-1}^{n+1}-0.5\frac{\Delta y}{\Delta X}\left[\begin{matrix}\begin{pmatrix}1.5\,u_{1j}^{n+1}-2\,u_{1j}^{n}\\+0.5\,u_{1j}^{n-1}\end{pmatrix}+\\\begin{pmatrix}1.5\,u_{1j-1}^{n+1}-2\,u_{1j-1}^{n}\\+0.5\,u_{1j-1}^{n-1}\end{pmatrix}\end{matrix}\right] \tag{51}$$

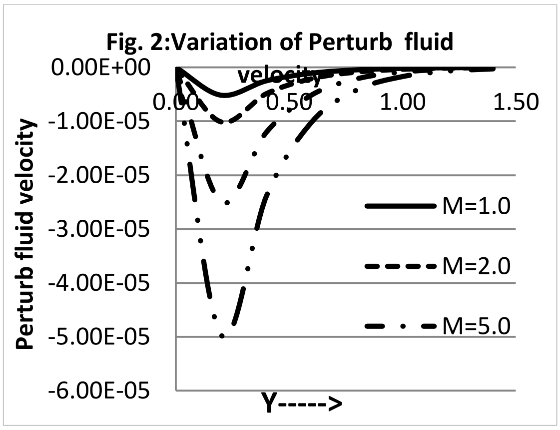
$$a_jW_{1_{j-1}}^{n+1}+b_jW_{1_j}^{n+1}+c_jW_{1_{j+1}}^{n+1}=d_j \tag{52}$$

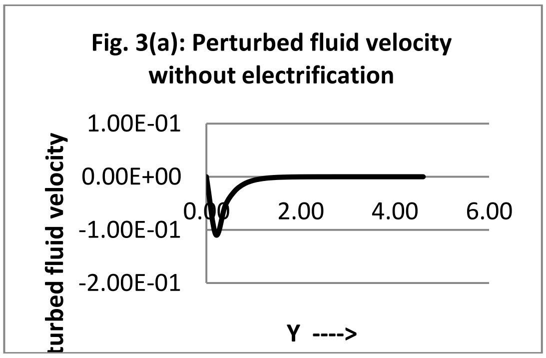
Where W_1 stands for perturbed flow

IV. DISCUSSION OF THE RESULTS

The following parametric values are used to study the different flow profiles as $\rho=0.913\ kg/m^3$; $\rho_p=8010\ kg/m^3$; $\alpha=0.1$; $D=50\mu m, 100\mu m$; $U=0.45\ m/\sec$; $L=0.044\ m$; $\mu=22.26\times10^{-6}$ kg/m sec; $\nu=2.43\times10^{-5}m^2/sec$

Fig -2 Shows the perturbed velocity (u_1) distribution against y for different value of M. The figure is Blasius type near the plate. The magnitude of u_1 increases with the increase of M.





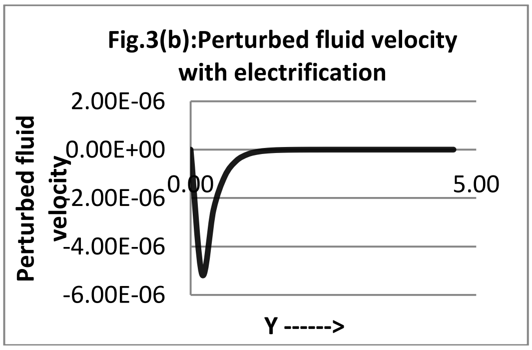


Fig- 3(a) and 3(b) shows the perturbed velocity profile u_1 without and with electrification of particles respectively. In both the cases the velocity distribution near the plate is of Blasius type and away from the plate it resembles with the distribution of plane free jet. It is observed that the numerical value of u_1 is greater in case of electrification of particles. Thus we conclude that the electrification of particles reduces the numerical value of u_1.

It is observed that the electrification of particles reduces the velocity.

NOMENCLATURE

(x, y)	$\rightarrow$	Space co-ordinates i.e. distance along the perpendicular to wall length(m)
(u, v)	$\rightarrow$	Velocity components for the fluid phase in x and y - directions respectively(m/s)
(u_p, v_p)	$\rightarrow$	Velocity components for the particle phase in x and y- directions respectively(m/s)
(υ, υ_p)	$\rightarrow$	Kinematic coefficient of viscosity of fluid and particle phase respectively
(ρ, ρ_p)	$\rightarrow$	Density of fluid and particle phase respectively(Kg/m^3)
(μ, μ_s)	$\rightarrow$	Coefficient of viscosity of fluid and particle phase respectively$(Kg/m.sec)$
τ_p	$\rightarrow$	Velocity equilibrium time
φ	$\rightarrow$	Volume fraction of Suspended particulate matter (SPM)
D	$\rightarrow$	Diameter of the particle(μm)
δ	$\rightarrow$	Boundary layer thickness
α	$\rightarrow$	Loading ratio
ϵ	$\rightarrow$	Diffusion parameter
e	$\rightarrow$	Charge per particle
M	$\rightarrow$	Electrification Parameter $\left(\frac{FL}{U^2}\left(\frac{e}{m}\right)E\right)$
F	$\rightarrow$	Friction parameter between the fluid and the particle $(F = 18\mu/\rho_p D^2)$
L	$\rightarrow$	Reference / Characteristic length
W	$\rightarrow$	Dummy variable
r_y	$\rightarrow$	Grid growth ratio
U	$\rightarrow$	Free stream velocity
η	$\rightarrow$	Edge of boundary layer

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