Well Test Analysis of Deforming Reservoirs by use of MDH Type Curves

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Abstract— *The objective of this study is to predict changes in formation thickness, permeability, and porosity, as a function of fluid pressure. The methodology depends on well test interpretation by type curve matching. Changes in formation permeability may lead to poor well performance. Compaction may lead to reduced well integrity, notably for vertical and deviated wells. Then, there may be additional relative movement between the well and the formation in comparison with conventional reservoirs. Use of well-known transformations leads to diffusivity equations of linear appearance. Many solutions are available. Corresponding pressure solutions may be obtained by an inverse transformation.We find that, for a deforming reservoir, analysis based on the method of permeability modulus gives the composite (sum) elastic modulus.By interpretation of MDH type curves for build-up tests, we quantify the values of the elasticity moduli for both the thickness and permeability.*

Keywords - *Deformable Reservoir Stress-sensitivity, Well Testing, Type Curves.*

I. INTRODUCTION

Most reservoirs may be thought about as rigid within engineering accuracy. For some reservoirs, this assumption may not be valid. Changes in formation permeability and thickness may lead to poor well performance and possible reduced well integrity. The basic assumptions of this study are: the volume of grains remains constant during deformation, no hysteresis and that all pressure dependent variables may be described by exponential functions of pressure. Then, the non-linearites show up as a sum of quadratic pressure gradient terms, characterized by a composite modulus, Matthews and Russell [1]. Use of a logarithmic transformation or the pseudo-pressure approach leads to a diffusivity equations of linear appearance.

The transformed equation, however, is still non-linear since the diffusivity depends on pressure. Hence, perturbation analysis may be necessary to improve accuracy. Raghavan et al. [2] proposed a well test model for pressure dependent rock and fluid properties by use of the pseudo-pressure approach. Their method allows for arbitrary pressure functions to characterize the rock properties. Pedrosa [3] assumed an exponential relationship between permeability and fluid pressure. Kikani and Pedrosa [4] matched the model predictions to real data. They showed that a constant dimensionless permeability modulus may be quantified by type curve analysis provided the initial permeability can be obtained. Their initial permeability value was estimated by use of a conventional two-permeability model. If a correct semi-log straight line may be identified, the initial permeability may be estimated by conventional methods. Then, well testing by type curve analysis alone becomes feasible. They warned against uncritical use of their method. Their study has inspired a lotof follow-up studies during the last decades. They argued that a first order perturbation, or even a zero order solution, may be of sufficient accuracy for many engineering calculations. As such, we investigate the zero and first order solutions only. If higher accuracy is required, second order perturbation techniques are available [4].

Zhang and Ambastha [5] investigated the validity of Kikani and Pedrosa [4] solutions by a finite difference numerical model. The authors found that the method works best for drawdown solutions, but can be used for small values of the dimensionless permeability modulus. They proposed a stepwise constant permeability modulus model to improve the match to published laboratory curves. Ozkan and Raghavan [6] discussed the application of Laplace transformations to facilitate solutions to the linear diffusivity equation. They presented many Laplace space solutions. These are available for stress-sensitive reservoirs with zero order accuracy.

Jelmert and Selseng [7] showed that the logarithmic transformation method and the pseudo-pressure approach is equivalent for exponential pressure functions. They used normalized permeability change as dependent variable. They pointed out that possible negative values are unphysical.

Once the value of an elastic modulus is known, the dynamic behaviour of the corresponding variable is also known [8]. The composite modulus technique can also be used for well performance calculations [9].

II. THEORY

We assume exponential variation with pressure. The elastic modulus of each variable shows up in the exponent as a factor to the pressure change.

$$x_{nj}\left(p\right)=e^{-\lambda_j\Delta p},\quad x_{nj}=\frac{x_j}{x_i},\ j=1,..,5$$
$$x_j=k\left(p\right),\rho\left(p\right),\mu\left(p\right),\varphi\left(p\right),h\left(p\right),$$
$$\lambda_j=\gamma,c,\upsilon,c_{ma},\xi \qquad (1)$$

In addition, we assume constant grain volume, the compaction is limited to the vertical direction and that flow due to changing thickness may be neglected and the validity of Darcy's law,Raghavan et al. [2]. Then:

$$h(p)=\frac{h_i\left(1-\varphi_i\right)}{1-\varphi\left(p\right)} \qquad (2)$$

and

$$u=-\frac{k\left(p\right)}{\mu\left(p\right)}\frac{dp}{dr} \qquad (3)$$

In Appendix A, we obtain the following simplified equation, eq.(A.12):

$$\frac{1}{r_D}\left\{\frac{\partial}{\partial r_D}\left(r_D\frac{\partial p_D}{\partial r_D}\right)-\tau_D r_D\left(\frac{\partial p_D}{\partial r_D}\right)^2\right\}=e^{\tau_D p_D}\frac{\partial p_D}{\partial t_D} \qquad (4)$$

An approximate solution to the above equation, which is still non-linear, may be obtained by the method of perturbations. Pedrosa [3] proposed the following substitution:

$$p_D\left(r_D,t_D\right)=-\frac{1}{\tau_D}\ln\left[1-\tau_D\eta\left(r_D,t_D\right)\right] \qquad (5)$$

First, we consider the line-source solution. Substitution ofeq.(5) into eq. (4), eq.(B.5), eq.(B.6) and eq.(B.7) yields:

$$\frac{\partial^2\eta}{\partial r_D^2}+\frac{1}{r_D}\frac{\partial\eta}{\partial r_D}=\frac{1}{1-\tau_D\eta}\frac{\partial\eta}{\partial t_D} \qquad (6)$$

$$\eta\left(r_D,0\right)=0 \qquad (7)$$

$$\lim_{r_D\to\infty}\eta\left(r_D,t_D\right)=0 \qquad (8)$$

$$\lim_{r_D\to 0}\left(r_D\frac{\partial\eta}{\partial r_D}\right)=-1,\qquad t_D\geq 0 \qquad (9)$$

This is the Pedrosa [3] problem, but with the composite modulus rather than the permeability modulus as perturbation parameter. As such, they share the samesolutions.

He [3] proposed the following perturbation scheme:

$$\eta=\eta_0+\tau_D\eta_1+\tau_D^2\eta_2+\tau_D^3\eta_3+\cdots \qquad (10)$$

Expanding the coefficient to the time derivative, eq.(6):

$$\frac{1}{1-\tau_D\eta}=1+\tau_D\eta+\tau_D^2\eta^2+\tau_D^3\eta^3+\cdots \qquad (11)$$

The perturbation scheme starts with the zero order solution, which has to satisfy eq.(7)- eq.(9), and eq.(12).

$$\frac{\partial^2\eta_0}{\partial r_D^2}+\frac{1}{r_D}\frac{\partial\eta_0}{\partial r_D}=\frac{\partial\eta_0}{\partial t_D} \qquad (12)$$

The solutions are, [3]:

$$\eta_0\left(r_D,t_D\right)=\frac{1}{2}E_1\left(\frac{r_D^2}{4t_D}\right) \qquad (13)$$

$$E_1\left(z\right)=\int_z^\infty\frac{e^{-x}}{x}dx \qquad (14)$$

$$\eta_1\left(r_D,t_D\right)=\frac{1}{2}E_1\left(2z\right)-\frac{1}{4}\left(1+e^{-z}\right)E_1\left(z\right) \qquad (15)$$

$$z=\frac{r_D^2}{4t_D} \qquad (16)$$

For build-up:

$$\eta_{0ws}=\eta_{0w}\left(t_{pD}+\Delta t_D\right)-\eta_{0w}\left(\Delta t_D\right) \qquad (17)$$

$$\eta_{1ws}=\eta_{1w}\left(t_p+\Delta t\right)-\eta_{1w}\left(\Delta t\right) \qquad (18)$$

which leads to:

$$\eta_{0ws}=\frac{1}{2}\left(E_1\left(\frac{1}{4\left(t_{pD}+\Delta t_D\right)}\right)-E_1\left(\frac{1}{4\Delta t_D}\right)\right) \qquad (19)$$

$$\eta_{1ws}=\frac{1}{2}E_1\left(\frac{1}{2\left(t_D+\Delta t_D\right)}\right)-\frac{1}{4}\left(1+e^{-\frac{1}{4\left(t_D+\Delta t_D\right)}}\right)E_1\left(\frac{1}{4\left(t_D+\Delta t_D\right)}\right)-\frac{1}{2}E_1\left(\frac{1}{2\Delta t_D}\right)+\frac{1}{4}\left(1+e^{-\frac{1}{4\Delta t_D}}\right)E_1\left(\frac{1}{4\Delta t_D}\right) \qquad (20)$$

The zero-order pressure solution becomes:

$$p_{0Dws}\left(\Delta t_D\right)=-\frac{1}{\tau_D}\ln\left(1-\frac{\tau_D}{2}E_1\left(\frac{1}{4\left(t_D+\Delta t_D\right)}\right)+\frac{\tau_D}{2}E_1\left(\frac{1}{4\Delta t_D}\right)\right) \qquad (21)$$

and the first order solution, which rather lengthy, may be obtained by:

$$p_{D1ws}=-\frac{1}{\tau_D}\ln\left[1-\tau_D\left(\eta_{0w}\left(t_D+\Delta t_D\right)-\eta_{0w}\left(\Delta t_D\right)\right.\right.-$$

$$\tau_D^2 \;\; \eta_{1w} \;\; t_p+\Delta t \;\; -\eta_{1w} \;\; \Delta t \qquad (22)$$

The zero and first order pressure solutions may be plotted against Δt_D in a log-log coordinate system, which is the MDH type curve, see Fig. 2. Kikani and Pedrosa[4]argued that the first order perturbation is multiplied by τ_D^2 , which usually assumes small values, hence the first order perturbation may be neglected for many engineering calculations.

The exponential integral has a logarithmic approximation, then eq.(21) will simplify to:

$$p_{0Dws} \;\; \Delta t_D \;\; =-\frac{1}{\tau_D}\ln\left(1-\frac{\tau_D}{2}\ln\frac{t_D+\Delta t_D}{\Delta t_D}\right) \qquad (23)$$

The traditional Horner equation is included in eq.(23) as a limiting behavior.

$$\lim_{\tau_D\to 0} \;\; p_D \;\; r_D, t_D \;\; =\frac{1}{2}\ln\frac{t_D+\Delta t_D}{\Delta t_D} \qquad (24)$$

The logarithmic derivative of eq.(23) becomes:

$$\frac{\partial p_{0wDs}}{\partial \ln \Delta t_D}=$$

$$-\frac{1}{2\left\{1-\frac{\tau_D}{2} \;\; \ln \;\; t_{Dp}+\Delta t_D \;\; -\ln \Delta t_D \;\; \right\}}\left(1-\frac{\Delta t_D}{t_D+\Delta t_D}\right) \qquad (25)$$

In many cases of practical interest, it is convenient to obtain Laplace space solutions and then do the inversion back to time domain by a numerical method.

Suppose the zero order unit step rate-solutions, $\bar{\eta}_{0wc}$ are known. Then, for each constant rate solution, the variable rate to the zero order solution can be obtained by, [10]:

$$\bar{\eta}_{0wv}=\frac{s\bar{\eta}_{0wc}+S \;\; 1-e^{st_D}}{s \;\; 1+s^2C_D\bar{\eta}_{0wc}+sC_D\bar{\eta}_{0wc}} \qquad (26)$$

The well may assume many shapes of simple geometry.

$$L^{-1} \;\; \bar{f} \;\; s \;\; 1-e^{st_D} \;\; \Big|_{t_D} = L^{-1} \;\; \bar{f} \;\; s \;\; \Big|_{t_D}$$

$$-L^{-1} \;\; \bar{f} \;\; s \;\; \Big|_{\Delta t_D} \qquad (27)$$

The equations may be numerically inverted back to time domain:

$$\eta_{w0}=L^{-1} \;\; \bar{\eta}_{w0} \qquad (28)$$

The time$t$ is the time since the start of the test withthe shut-in time Δt included. With known zero-order solution, one may improve the accuracy by perturbations as explained above.

III.WELL TEST INTERPRETATION

Kikani and Pedrosa [4] showed how to determine the permeability modulus by type curve matching. We use the same technique to obtain the composite modulus. If the field curve and the type curve can be matched, this observation supports the assumption of exponential variation with pressure.

Assumption: No wellbore storage or skin.The initial permeability-thickness product, k_ih_i may be estimated from the pressure match or other sources. Suppose it can be obtained from the pressure match, then:

$$k_ih_i \;\; _{ic}=\frac{q_{sf}\mu_i}{2\pi}\left(\frac{\Delta p}{p_D}\right)_{Mp} \qquad (29)$$

This match, however, may be hard to achieve. If the permeability-thickness product is available from another source, the dimensionless elasticity modulus, τ_D is available as a parameter from the matched type curve, Fig. 1. From eq.(A4) we have:

$$\tau_{Tc}=\gamma+\xi+c_1-\upsilon \qquad (30)$$

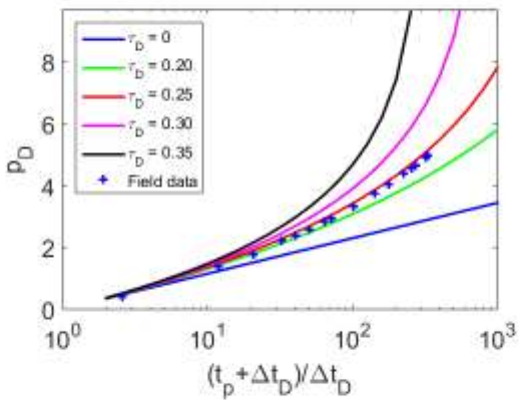

Fig. 1 Comparison of model and real data

From the time match, we have:

$$\Delta t_{DMp}=\frac{k_i\Delta t_{Mp}}{\varphi_i \;\; \xi+c_1+c_{ma} \;\; \mu_i r_w^2} \qquad (31)$$

Which gives:

$$\xi_{Tc}=\frac{k_i}{\varphi_i\mu_i r_w^2}\left(\frac{\Delta t_{Mp}}{\Delta t_{DMp}}\right)- \;\; c_1+c_{ma} \qquad (32)$$

The field curve corresponding to Fig. 1 has the disadvantage that the Horner ratio is dimensionless. AMDH type curve,on the other hand, permits a dimensional horizontal axis in the field curve. Then, additional information becomes available.

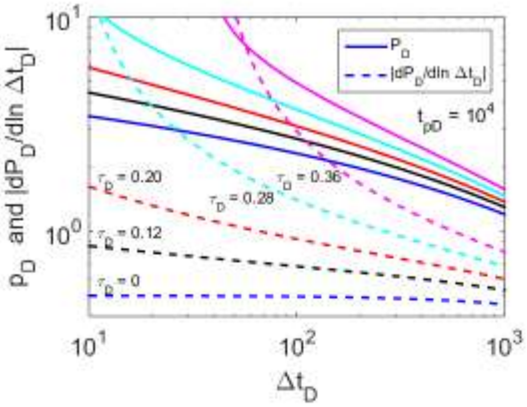

Fig.2 Dimensionless pressure and pressure derivative, MDH type curve

Combination of eq.(1) and eq.(32) leads to:

$$h_n = \frac{h}{h_i} = e^{-\xi_{Tc}\Delta p_{ws\ obs}} \quad (33)$$

Or

$$h_n = e^{-\xi_{D\ Tc}\, p_{D\ ws\ Tc}} \quad (35)$$

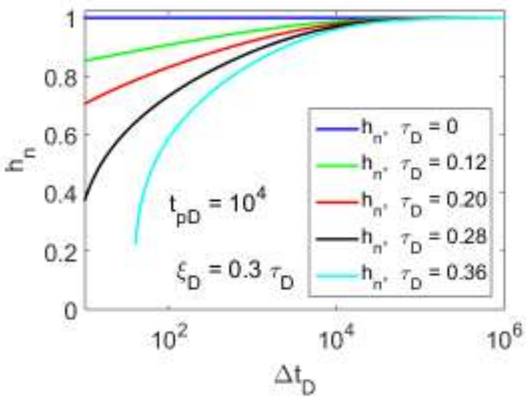

Fig.3Normalized thickness as function of dimensionlesstime

The normalized thickness may be plotted as afunction of pressure or time. The latter has been plotted in Fig. 3.But from eq.(30):

$$\tau_{Tc} = \gamma + \xi_{Tc} + c_1 - \upsilon \quad (36)$$

Then

$$\gamma_{Tc} = \tau_{Tc} - \xi_{Tc} + c_1 - \upsilon \quad (37)$$

The estimated normalized permeability becomes:

$$k_n = \frac{k}{k_i} = e^{-\gamma_{Tc}\Delta p_{ws\ obs}} \quad (38)$$

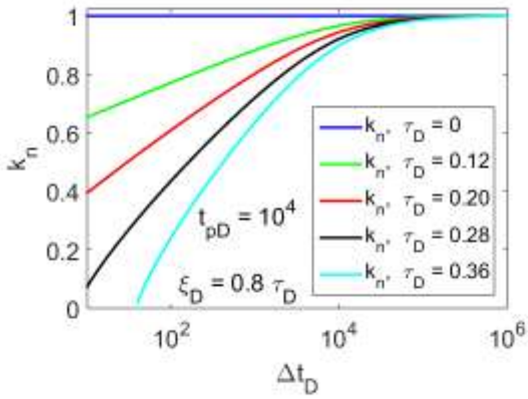

Fig.4Normalized permeability as function of dimensionlesstime

IV. VERIFICATION OF NUMERICAL RESULTS

We found that the generalized equations for deformable reservoirs are equivalent to those derived by Kikani and Pedros [4]. Hence, their results may be used as benchmarks. Our figures 1, 2 and 5 were compared against their figures 5, 4 and 7, respectively and we conclude that the results agrees well within the accuracy of visual inspection.

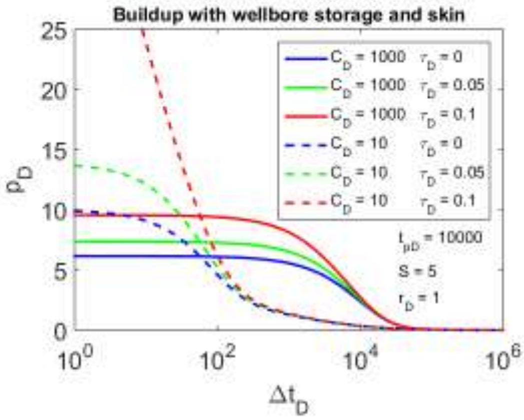

Fig. 5: Effect of wellbore storage and skin

V. CONCLUSIONS

A well-known well test interpretation technique for stress-sensitive reservoirs has been generalized to account for thickness, porosity and permeability changes.

The elastic moduli forpermeability and thickness may be obtained. With known elastic moduli, the dynamic behavior of the corresponding variables are also known.

For a deforming reservoir, analysis based on Pedrosa's method gives the total elastic modulus rather than the permeability modulus.

The proposed methodology depends on known initial condition for the permeability thickness product. If a semi-log straight line shows up in a MDH- or Horner plot, the initial condition may be established by well test interpretation.

NOMENCLATURE

B Formation volume factor
C Wellbore storage constant, Pa^{-1}
c Compressibilty, Pa^{-1}
T_n Normalized transmissibility function, given by eq.(A3)
ΔT_n Change in normalized transmissibility from the reference value, $\Delta T_n = 1 - T_n$
h Thickness, m
k_i Initial permeability, m^2
p Fluid pressure, Pa
q Flow rate, Sm^3/s
r Radial distance, m
r_D Dimensionless distance, $r_D = r/r_w$
S Mechanical skin factor
s Laplace variable
t_D Dimensionless time
Δt_D Dimensionless shut-in time

Greek letters
μ Viscosity, $Pa\ s$
γ Permeability modulus, Pa^{-1}
φ Porosity
τ Composite modulus, eq.(A4)
υ Viscosity modulus, Pa^{-1}
ξ Thickness modulus, Pa^{-1}
η Transformed variable, eq.(5)
ρ Density, $kg\ m^{-3}$

Indices
c Constant rate condition
ma Matrix
s Shut-in
Tc Value obtained by type curve analysis
v Variable rate conditions
w Variable evaluated at r_w
l Fluid
0,1 Order of perturbation

APPENDIX A

The diffusivity equation for consolidated media is [2]:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(\frac{\rho_i k_i h_i}{\mu_i}\frac{\rho_n\ p\ k_n\ p\ h_n\ p}{\mu_n\ p} r\frac{\partial \Delta p}{\partial r}\right) = \frac{\partial}{\partial t}\ h\ p\ \rho\ p\ \varphi\ p \qquad \text{(A1)}$$

Index n denotes normalized to initial condition. We define a composite variable, $T_n\ p$ and assume exponential variation with pressure of each individual variable. Then:

$$T_n\ p = \frac{\rho_n\ p\ k_n\ p\ h_n\ p}{\mu_n\ p} \qquad \text{(A2)}$$

$$T_n\ p = e^{-\ c_1+\gamma+\xi-\upsilon\ \ p_i-p} = e^{-\tau\ p_i-p} \qquad \text{(A3)}$$

Integration by parts gives composite elastic modulus simply as the sum of the individual moduli:

$$\tau = c_1 + \gamma + \xi - \upsilon \qquad \text{(A4)}$$

Substitution into eq.(A1) yields:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(e^{-\tau\Delta p} r\frac{\partial \Delta p}{\partial r}\right) = \frac{\varphi_i \mu_i}{k_i} e^{-\ \xi+c_1+c_{ma}\ \Delta p}\ \xi + c_1 + c_{ma}\ \frac{\partial \Delta p}{\partial t} \qquad \text{(A5)}$$

After some manipulations, we obtain:

$$\frac{1}{r}\left\{\frac{\partial}{\partial r}\left(r\frac{\partial \Delta p}{\partial r}\right) - \tau r\left(\frac{\partial \Delta p}{\partial r}\right)^2\right\} = \frac{\varphi_i \mu_i}{k_i} e^{\ \gamma+\xi-\upsilon+c_1-\ \xi+c_1+c_{ma}\ \ \Delta p}\ \xi + c_1 + c_{ma}\ \frac{\partial \Delta p}{\partial t} \qquad \text{(A6)}$$

Assumption: $\gamma + \xi - \upsilon + c_l >> \xi + c_1 + c_{ma}$

Then, the above equation will reduce to:

$$\frac{1}{r}\left\{\frac{\partial}{\partial r}\left(r\frac{\partial \Delta p}{\partial r}\right) - \tau r\left(\frac{\partial \Delta p}{\partial r}\right)^2\right\} = \frac{\varphi_i \mu_i}{k_i} e^{\tau\Delta p}\ \xi + c_1 + c_{ma}\ \frac{\partial \Delta p}{\partial t} \qquad \text{(A7)}$$

$$r_D = \frac{r}{r_w} \qquad \text{(A8)}$$

$$t_D = \frac{k_i t}{\varphi_i\ \ \xi + c_1 + c_{ma}\ \ \mu_i r_w^2} \qquad \text{(A9)}$$

$$p_D = \frac{2\pi k_i h_i}{q_{sc}\mu_i}\Delta p \qquad \text{(A10)}$$

$$\tau_D = \frac{q_{sc}\mu_i \tau}{2\pi k_i h_i} \qquad \text{(A11)}$$

Substitution of eq.(A8)-(A11) into eq. (A7) yields:

$$\frac{1}{r_D}\left\{\frac{\partial}{\partial r_D}\left(r_D\frac{\partial p_D}{\partial r_D}\right)-\tau_D r_D\left(\frac{\partial p_D}{\partial r_D}\right)^2\right\}$$
$$=e^{\tau_D p_D}\frac{\partial p_D}{\partial t_D} \qquad \text{(A12)}$$

APPENDIX B

We consider a line-source well in an infinite reservoir. The initial condition is:

$$\Delta p=0 \qquad \text{(B1)}$$

The boundary conditions are:

External:

$$\lim_{r\to\infty}\Delta p\ \ r,t\ \ =0 \qquad \text{(B2)}$$

Internal:

$$q_{sc}=\frac{2\pi\rho_i k_i h_i r}{\mu_i}e^{-\tau\ \ p_i-p}\ \frac{\partial p\ \ r_w}{\partial r} \qquad \text{(B3)}$$

Line-source boundary condition:

$$\lim_{r\to 0}\left(e^{-\tau\Delta p}r\frac{\partial p}{\partial r}\right)=\frac{q_{sc}\mu_i}{2\pi k_i h_i} \qquad \text{(B4)}$$

Substitution of the dimensionless variables into eq.(B1), eq.(B2) and eq.(B4) yields:

$$p_D=0 \qquad \text{(B5)}$$

$$\lim_{r\to\infty}\Delta p\ \ r,t\ \ =0 \qquad \text{(B6)}$$

$$\lim_{r\to 0}\left(e^{-\tau_D p_D}r_D\frac{\partial p_D}{\partial r_D}\right)=-1 \qquad \text{(B7)}$$

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