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Optimal Control of Mathematical Model for COVID-19 with Quarantine and Isolation

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Abstract — *This study discusses the solution to the optimal control problem of the COVID-19 model with preventive action through education and treatment of infected individuals. In this model, the population is divided into seven subpopulations: subpopulation of susceptible, exposed, symptomatic, asymptomatic, quarantine, isolated, and recovered. Optimal control is obtained using the Pontryagin minimum principle and solved numerically using the Forward-Backward Sweep method. Furthermore, given control measures can minimize the number of subpopulations: exposed, symptomatic, and asymptomatic, and significant costs associated with control.*

Keywords — *COVID-19 model, Optimal control, Pontryagin minimum principle, Forward-Backward Sweep method.*

I. INTRODUCTION

COVID-19 first appeared and was identified in Wuhan-Hubei Province, China, around December 2019 [1]. Furthermore, the virus spreads to various countries rapidly through individuals who have had a history of travel to Wuhan [2]–[5]. Before being reported and informed to the public, a doctor named Li Wenliang had provided information about the emergence of this virus because seven patients from the local seafood market were diagnosed with a SARS-like disease and were quarantined in a hospital [6].

How to control every time there is an outbreak or the emergence of an infection in a region or country in the absence of vaccines or treatment, isolation, and individual quarantine are the most effective ways [7]. According to [8], quarantine is defined as restricting activities or segregating susceptible individuals as long as there is no essential need to leave the house. Then, individuals who have a history of contact with individuals infected with COVID-19 or have a history of traveling to an area where local transmission has occurred and separate themselves by staying at home during the incubation period (2 weeks) are also included in the quarantine group. Furthermore, isolation is defined as separating a sick or infected individual from other individuals directly in the hospital or at home (self-isolation) with medical personnel monitoring. WHO provides several appeals to control COVID-19, namely social distancing, masks in public places, tracing followed by quarantine of potentially affected individuals, and isolation of infected individuals in hospitals or independently. Furthermore, such control measures to be implemented by the wider community require education to be provided (such as social media, TV media, online media, billboards or banners, etc.). Efforts to treat infected individuals also need to be given because they can reduce the number of individuals infected with COVID-19.

Several studies related to the spread of disease, for example, research on the Coronavirus which resulted in SARS [9] and MERS [7], [10]. Then the Coronavirus developed into a virus known as the COVID-19 virus and became a hot topic in 2020.

Soewono [11] models the initial spread of COVID-19 by applying the *SEIR* model, which consists of four subpopulations: S (susceptible), E (exposed), I (infected with symptoms/ symptomatic), and R (recovered). Furthermore, Belgaid et al. [12] added subpopulation A (infected without symptoms/ asymptomatic). The population is divided into five subpopulations, namely S, E, I, A, and R, by showing symptoms and some who do not show symptoms. In another study, Zeb et al. [13] added subpopulation H (isolation). The population is divided into five subpopulations, namely S, E, I, H, and R. This is based on the latest information that infected individuals will spread to surrounding individuals because they are not given isolation measures. A study on COVID-19 was also carried out by Jia et al. [14] involving quarantine (Q) and isolation (H) subpopulations so that the model presented divides the population into seven subpopulations: S, E, I, A, Q, H, and R. The model made is also based on the latest information from WHO, that susceptible individual should be quarantined first to reduce further spread.

Furthermore, several studies related to control, such as Olaniyi et al. [15], construct S, E, I, A, H, R, and D (mortality subpopulation) models. The model adds two control, namely prevention through education given to susceptible subpopulations and improving management of isolated individual care given to isolation subpopulations. Olaniyi et al. aim to minimize the number of subpopulations I, A, and H. Then, Deressa and Duressa [16] with S, E, I, A, H, and R models propose two control measures such as Olaniyi et al. and adding one more control, namely implementing what is intended to protect oneself (such as social distancing, tracing, using masks, etc.). The study from Deressa and Duressa has the same goal as Olaniyi et al. and goals to minimize E subpopulations.

In this study, a COVID-19 model will be constructed by combining the research of Belgaid et al. [12], Zeb et al. [13], and Jia et al. [14]. In this study, the population was divided into subpopulations: S, E, I, A, Q, H, and R. Furthermore, two control measures were added to the constructed model, namely 1) preventive action through education (such as social distancing, using masks, clean life, etc.), and 2) there are efforts to treat infected individuals. Solves optimal control problems to minimize exposed, symptomatic and asymptomatic subpopulations and minimize associated costs. Furthermore, optimal control is solved by Pontryagin's minimum principle. The numerical simulation will be performed using the Forward-Backward Sweep method with the Matlab R2017a software in the final section.

II. METHODS

The method in this study can be seen in the flowchart below

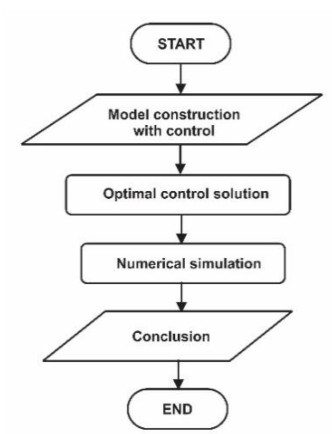

Fig 1: Flowchart for method in this study

The problem of optimal control here is determining whether to maximize or minimize the objective function. In optimal control problems with systems of ordinary differential equations, the control variable is denoted $u(t)$, , and the state variable is denoted $x(t)$. The state of a continuous system is called the constraint function and is expressed in terms of

$$\frac{d\vec{x}}{dt} = g\left(t, \vec{u}(t), \vec{x}(t)\right).$$

The objective function is influenced by $u(t)$ and $x(t)$ with the following general form

$$Z\left(\vec{u}\right) = \int_{t_0}^{t_N} h\left(t, \vec{u}(t), \vec{x}(t)\right) dt,$$

With boundary conditions $x(t_0) = x_0$ and $x(t_N)$ free.

The control variable that optimizes Z is denoted $u^*(t)$. Then $u^*(t)$ is substituted into the state $x(t)$ equation to obtain the optimal state denoted $x^*(t)$ [17].

The Pontryagin principle is a necessary condition to obtain an optimal solution. This principle changes the optimal control problem that minimizes or maximizes the objective function to minimize or maximize the Hamilton function. L. S. Pontryagin developed this principle in 1950. This study uses the Pontryagin minimum principle. This principle states the conditions necessary for a control $u(t)$ to optimize the objective function. This condition is determined by constructing a function called the Hamilton function by introducing a new variable called the costate variable and denoted $\lambda(t)$. Hamilton's function is as follows

$$\mathrm{H} = h\left(t, u(t), x(t)\right) + \sum_{i=1}^{n} \lambda_i(t)\, g_i\left(t, \vec{u}(t), \vec{x}(t)\right),$$

with h is integral in the objective function of the equation and g is the state equation of the right-hand side, $i = 1, 2, ...n.$

If $\vec{u}^*(t)$ and $\vec{x}^*(t)$ are the values that optimize the problem and, then the variable costate $\vec{\lambda}(t)$ will exist if

$$\mathrm{H}\left(t, \vec{\lambda}(t), \vec{u}^*(t), \vec{x}^*(t)\right) \le \mathrm{H}\left(t, \vec{\lambda}(t), \vec{u}(t), \vec{x}^*(t)\right).$$

This condition states that the optimal control $\vec{u}^*(t)$ must be determined, which minimizes the Hamiltonian function H at time t. Furthermore, if the Hamilton function can be derived for $\vec{u}(t)$, the condition if

$$\frac{\partial \mathrm{H}}{\partial \vec{u}} = 0.$$

The step to determine whether the control obtained is the minimum or maximum control can be done by examining the second derivative of Hamilton's function against $\vec{u}(t)$. If

$$\frac{\partial^2 \mathrm{H}}{\partial \vec{u}^2} = 0.$$

Then, the control problem that is solved is the minimum control problem, and vice versa.

In addition to the control $u(t)$ variable, the Hamilton function contains the state variables $x(t)$ and costate $\lambda(t)$. Furthermore, the state equation $\vec{x}'$ is as follows

$$\vec{x}' = \frac{\partial \mathrm{H}}{\partial \lambda_i} = g_i\left(t, \vec{u}(t), \vec{x}(t)\right),$$

and the costate equation $\vec{\lambda}'$ can be expressed as

$$\vec{\lambda}' = \frac{\partial \lambda_i}{\partial t} = -\frac{\partial \mathrm{H}}{\partial x_i}.$$

Suppose the initial value $\vec{x}(0)$ and the final value $\vec{x}(t_N)$ are given. In that case, we can immediately determine the value of the derivative $\vec{x}'$ and $\vec{\lambda}'$. However, if the final condition $\vec{x}(t_N)$ is not given. A condition called the transverse condition is used, namely $\vec{\lambda}(t_N)=0,$ the final condition.

Based on this description, Pontryagin's minimum principles include the following conditions.

a. $\mathrm{H}\left(t,\vec{\lambda}(t),\vec{u}^*(t),\vec{x}^*(t)\right)\le \mathrm{H}\left(t,\vec{\lambda}(t),\vec{u}(t),\vec{x}^*(t)\right),$

b. $\dfrac{d\vec{x}}{dt}=\dfrac{\partial \mathrm{H}}{\partial \vec{\lambda}},$

c. $\dfrac{d\vec{\lambda}}{dt}=-\dfrac{\partial \mathrm{H}}{\partial \vec{x}},$

d. $\vec{\lambda}(T)=0.$

The constraint is given $a\le \vec{u}\le b$ on the control variable $\vec{u}(t)$. Then the condition $\dfrac{\partial \mathrm{H}}{\partial \vec{u}}=0$ changes to

$$\vec{u}^*=\begin{cases}\vec{u}=a, & \text{jika } \dfrac{\partial \mathrm{H}}{\partial \vec{u}}<0,\\ a\le \vec{u}\le b, & \text{jika } \dfrac{\partial \mathrm{H}}{\partial \vec{u}}=0,\\ \vec{u}=b, & \text{jika } \dfrac{\partial \mathrm{H}}{\partial \vec{u}}>0.\end{cases}$$

Method of Forward-Backward Sweep is a method of numeric to solve the optimal control problem [18]. The Forward-Backward Sweep method algorithm is as follows

1. Specifies the initial guess $\vec{u}$.
2. Using the initial condition $\vec{x}(0)=\vec{x}_0$ and the value $\vec{u}$ to solve the state $\vec{x}$ equation.
3. Using the transverse condition $\vec{\lambda}(t_1)=0,$, the initial values $\vec{u}$ and $\vec{x}$ to solve the costate equation.
4. Updates control value $\vec{u}$ by entering the values $\vec{x}$ and $\vec{\lambda}$ into the equation for the characteristics of the optimal control.

Suppose the error value of each state, costate variable, and the control in the current iteration with the previous iteration is very small than the given error tolerance value. In that case, the value of each variable in the current iteration is a solution. However, if the error value is not very small, then go back to step two.

III. RESULTS

The results of the discussion are divided into several sub-sections as follows:

A. Optimal control problem formulation

The control variables assigned to the COVID-19 model consisted of preventive action through education u_1, and the presence of treat for an infected individual u_2. Hence, the system of equations as follows:

$$\begin{aligned}
\frac{dS}{dt}&=\Lambda-(1-u_1)S(\beta_1E+\beta_2I+\beta_3A)-S(\mu+q_1),\\
\frac{dE}{dt}&=(1-u_1)S(\beta_1E+\beta_2I+\beta_3A)-E(\mu+\sigma+q_2),\\
\frac{dI}{dt}&=\sigma\omega E+\theta A-I(\gamma_1+\mu+d),\\
\frac{dA}{dt}&=(1-\omega)\sigma E-A(\gamma_3+\theta+\mu),\\
\frac{dQ}{dt}&=q_1S+q_2E-Q(\mu+\gamma_2+r_2),\\
\frac{dH}{dt}&=\gamma_1I+\gamma_2Q+\gamma_3A-H(\mu+r_1),\\
\frac{dR}{dt}&=r_1H+r_2Q-\mu R.
\end{aligned}\tag{1}$$

The u_1 control variable is targeted to prevent or reduce transmission of COVID-19. Then $(1-u_1)$ causes the rate of the uneducated susceptible subpopulation. Furthermore, a control variable u_2 was assigned to reduce the number of symptomatic and asymptomatic subpopulations.

This study aims to minimize subpopulations: exposed, symptomatic, asymptomatic, and minimize control costs. The optimal control problem is expressed in terms of the objective function or objective function, minimized as follows:

$$Z(u_1,u_2)=\int_0^{t_N}\left(E+I+A+\frac{1}{2}\left(C_1u_1^2+C_2u_2^2\right)\right)dt,$$

A system of equations is a constraint. Then C_1 is the weight of control costs in the form of preventive action through education, and C_2 is the weight of control costs to treat infected individuals. Furthermore, the optimal control will be determined u_1^* and u_2^* which fulfill the objective function, namely

$$Z\left(u_1^*,u_2^*\right)=\min\left\{J(u_1,u_2)\,|\,u_1,u_2\in v\right\},$$

with $v=\left\{(u_1(t),u_2(t)):0\le u_1(t)\le 1, 0\le u_2(t)\le 1\right\}.$

B. Optimal control solution

Based on the objective function and system constraints of equations, the first step to solving the optimal control problem is to form the Hamilton function.

$$\mathrm{H}=E+I+A+\frac{1}{2}\left(C_1u_1^2+C_2u_2^2\right)+\sum_{i=1}^{7}\lambda_i g_i(t,\vec{u},\vec{x}).$$

From the Pontryagin minimum principle, the Hamilton function will achieve the optimal solution if it satisfies the state equation, costate equation, and stationary conditions. The state equation is obtained by deriving the Hamilton function for each costate variable as follows:

$$\frac{dS}{dt} = \Lambda - (1-u_1) S(\beta_1 E + \beta_2 I + \beta_3 A) - S(\mu + q_1),$$
$$\frac{dE}{dt} = (1-u_1) S(\beta_1 E + \beta_2 I + \beta_3 A) - E(\mu + \sigma + q_2),$$
$$\frac{dI}{dt} = \sigma\omega E + \theta A - I(\gamma_1 + \mu + d),$$
$$\frac{dA}{dt} = (1-\omega)\sigma E - A(\gamma_3 + \theta + \mu),$$
$$\frac{dQ}{dt} = q_1 S + q_2 E - Q(\mu + \gamma_2 + r_2),$$
$$\frac{dH}{dt} = \gamma_1 I + \gamma_2 Q + \gamma_3 A - H(\mu + r_1),$$
$$\frac{dR}{dt} = r_1 H + r_2 Q - \mu R.$$

The costate equation is the negative value of the derivative of the Hamilton function for each state variable as follows:

$$\frac{d\lambda_1}{dt} = -\frac{d\mathrm{H}}{dS} = KZ_1 + \lambda_1(\mu + q_1) - \lambda_5 q_1,$$
$$\frac{d\lambda_2}{dt} = -\frac{d\mathrm{H}}{dE} = K\beta_1 S + \lambda_2 Z_2 - \lambda_3\sigma\omega - \lambda_4(1-\omega)\sigma - \lambda_5 q_2 - 1,$$
$$\frac{d\lambda_3}{dt} = -\frac{d\mathrm{H}}{dI} = K\beta_2 S + \lambda_3 Z_3 - \lambda_6(\gamma_1 + u_2) - 1,$$
$$\frac{d\lambda_4}{dt} = -\frac{d\mathrm{H}}{dA} = K\beta_3 S - \lambda_3\theta + \lambda_4 Z_4 - \lambda_6(\gamma_3 + u_2) - 1,$$
$$\frac{d\lambda_5}{dt} = -\frac{d\mathrm{H}}{dQ} = \lambda_5(\mu + \gamma_2 + r_2) - \lambda_6\gamma_2 - \lambda_7 r_2,$$
$$\frac{d\lambda_6}{dt} = -\frac{d\mathrm{H}}{dH} = (\lambda_6 - \lambda_7) r_1 + \lambda_6\mu,$$
$$\frac{d\lambda_7}{dt} = -\frac{d\mathrm{H}}{dR} = \lambda_7\mu,$$

where $Z_1 = (\beta_1 E + \beta_2 I + \beta_3 A)$, $Z_2 = (\mu + \sigma + q_2)$, $Z_3 = (\gamma_1 + u_2 + \mu + d)$, $Z_4 = (\gamma_3 + u_2 + \theta + \mu)$, and $K = (\lambda_1 - \lambda_2)(1-u_1)$.

Then, the transverse condition $\lambda_j(t_N) = 0$, where $j = 1, 2, \ldots 7$.

Thus, the optimal controls u_1^* and u_2^* can be expressed as

$$u_1^* = \max\left\{0, \min\left(\frac{S^*(\beta_1 E^* + \beta_2 I^* + \beta_3 A^*)(\lambda_2 - \lambda_1)}{C_1}, 1\right)\right\},$$

and

$$u_2^* = \max\left\{0, \min\left(\frac{I^*(\lambda_3 - \lambda_6) + A^*(\lambda_4 - \lambda_6)}{C_2}, 1\right)\right\}.$$

Substitute the optimal control variables (u_1^* and u_2^*) into the state and costate equation to get the optimal system.

C. Numerical simulation

The parameter values are used in Table 1 for numerical simulations.

Table 1: Parameters Value

Parameter	Parameter Value	Source
Λ	1.685	Assumed
μ	0.000039139	[19]
σ	0.0196	Assumed
θ	0.01	[19]
ω	0.4	[19]
d	0.087	[20]
q_1	0.09	[21]
q_2	0.1	[21]
β_1	0.01	[21]
β_2	0.1	Assumed
β_3	0.1	Assumed

Furthermore, the weights for each strategy in the numerical simulation use $C_1 = 0.025$, and $C_2 = 0.25$ respectively, and the simulation interval $t \in [0, 100]$.

a) Strategy simulation I

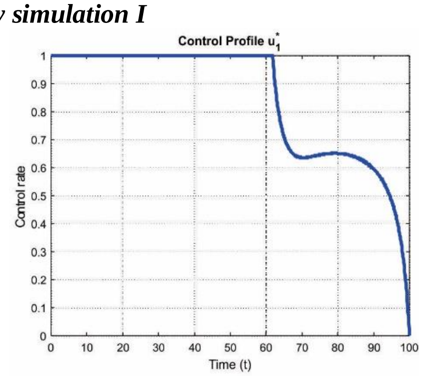

Fig 2: Control profile u_1

Fig 2 is a control profile for preventive action through education only and aims to reduce cases of COVID-19 as long as $t = 100$. This control is given at one or maximum at the start of the period up to 61.8, and decreases slowly towards zero. Control is stopped at the end of the period, which means there is no more preventive action control through the education provided. Furthermore, the effects of a given control are presented in Fig 3.

Fig 3(a) shows that the existence of strategy I add to the susceptible subpopulation. Then, this happens because susceptible individuals who have received education will always be careful, such as interacting outside the home. The impact of implementing strategy I make the subpopulation: exposed, symptomatic, and asymptomatic less. This can be seen in Fig. 3 (b), 3 (c), and 3 (d), respectively.

The strategy I implementation can increase the quarantine subpopulation, as shown in Fig 3(e). The existence of educational controls makes each choose to quarantine at home because they are aware of the dangers of the COVID-19 virus so that the number of infections is reduced. The number of subpopulations: symptomatic and asymptomatic has decreased, resulting in a reduced number of isolated subpopulations, as shown in Fig 3(f). Furthermore, the subpopulation recovered increased, as shown in Fig 3(g).

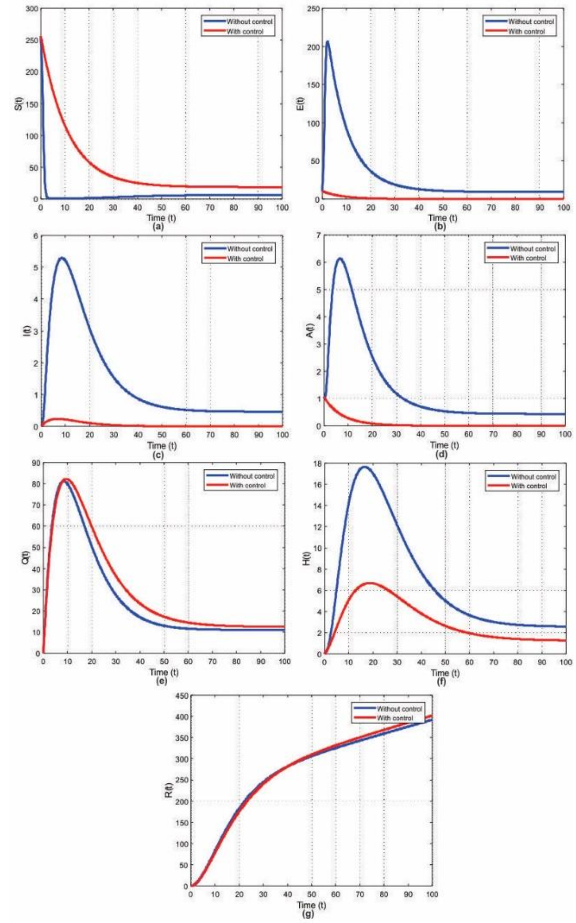

Fig 3: Optimal control simulation results with control u_1

b) Strategy simulation II

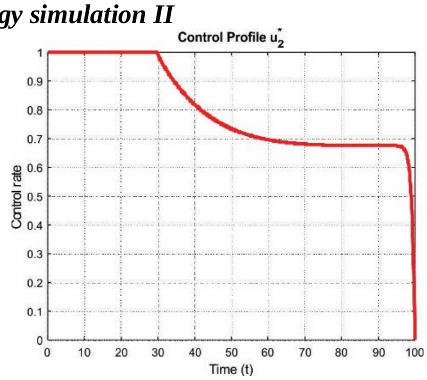

Fig 4: Control profile u_2

Fig 4 shows a control profile for the treatment of infected individuals to reduce COVID-19 cases. This control is given at one or maximum from the start of the period to $t = 29.8$ and decreases slowly towards zero. Control is stopped at the end of the period, which means that no more control was given to the treatment of the infected individual. Furthermore, the effects of the given controls are presented in Fig 5.

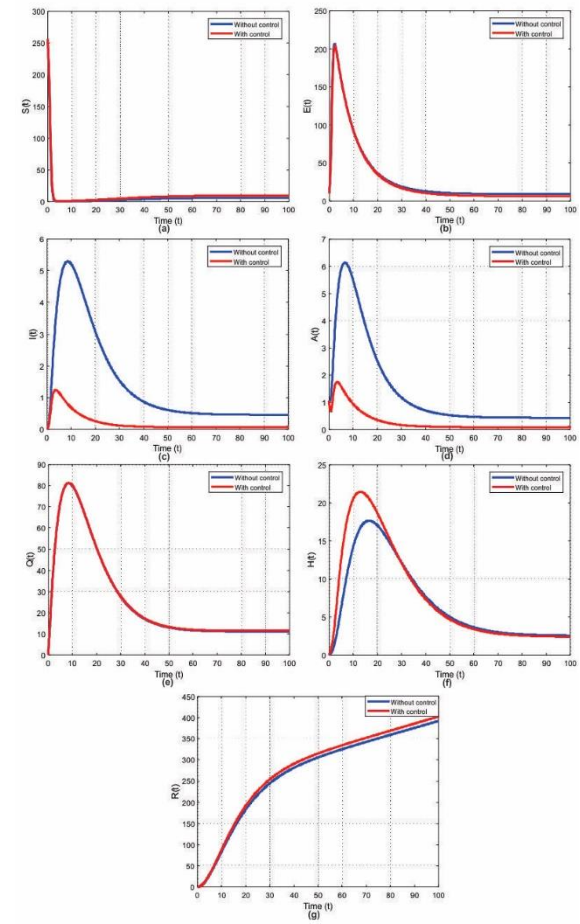

Fig 5: Optimal control simulation results with control u_2

Fig 5(a) shows that the existence of strategy II can increase the susceptible subpopulation, even if only slightly. Then, this happens because some individuals are afraid or aware of the COVID-19 virus, so they don't always travel outside the home. Implementing strategy II on reducing the exposed subpopulation is slightly shown in Fig 5(b).

Implementation of strategy II was able to reduce the symptomatic and asymptomatic infected subpopulations as seen in Fig 5(c) and 5(d). Then the slight change of the susceptible and exposed subpopulations has an impact on changing the quarantine subpopulation, as seen in Fig 5(e). Furthermore, Fig 5(f) shows that the control efforts to treat infected individuals have an impact on the number of isolation subpopulations. In the end, the isolation subpopulation has decreased slightly. Then, this suggests that implementing strategy II can reduce subpopulation numbers: exposed, symptomatic, and asymptomatic, but there is a burden as the isolation subpopulation increases. Furthermore, increasing subpopulations: quarantine and isolation made recovered subpopulations increase, as shown in Fig 5(g).

c) Strategy simulation III

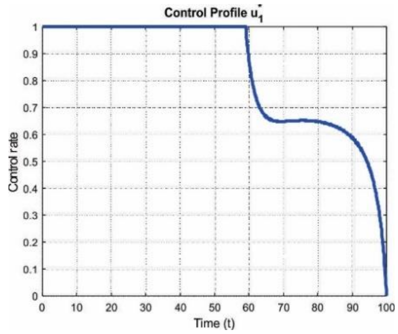

Fig 6: Control profile u_1 on combination strategy u_1 and u_2

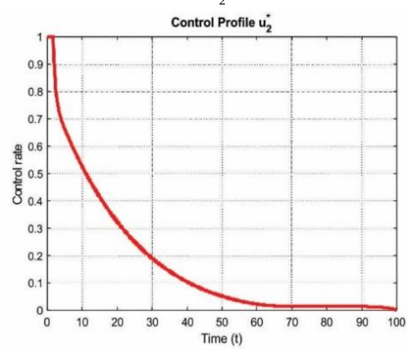

Fig 7: Control profile u_2 on combination strategy u_1 and u_2

Control combination between preventive action control through education and the presence of treatment for infected individuals can reduce cases of COVID-19 as long as $t = 100.$ Fig 6 shows a control profile for preventive action through education. Control should be given a maximum from the start to $t = 59,$ and decreases slowly towards zero. Next, Fig 7 shows a control profile for the treatment effort of an infected individual. Control is given one or maximum from the start of the period up to $t = 1.6,$ and decreases slowly towards zero at the end of the period. Control is stopped at the end of the period, meaning that there are no more preventive control measures through education and treatment efforts for infected individuals are given.

Fig 8(a) shows that the presence of strategy III can increase the number of susceptible subpopulations. Then, this happens because susceptible individuals who have received education will always be careful, such as interacting outside the home. The impact of implementing strategy III makes the exposed subpopulation reduced, as shown in Fig 8(b).

Strategy III implementation can reduce subpopulations (symptomatic and asymptomatic) more maximally than strategies I and II, as seen in Fig 8(c) and Fig 8(d). Combining preventive action control through education and treatment for infected individuals makes each individual choose home quarantine to increase the quarantine subpopulation, as shown in Fig 8(e). The subpopulation (symptomatic and asymptomatic) has decreased due to education, and continued treatment of the infected individual will reduce the subpopulation: symptomatic and asymptomatic. The reduction in the subpopulation (symptomatic and asymptomatic) results in a reduction in the isolation subpopulation, as shown in Fig 8 (f). Furthermore, the subpopulation recovered increased, as shown in Fig 8(g).

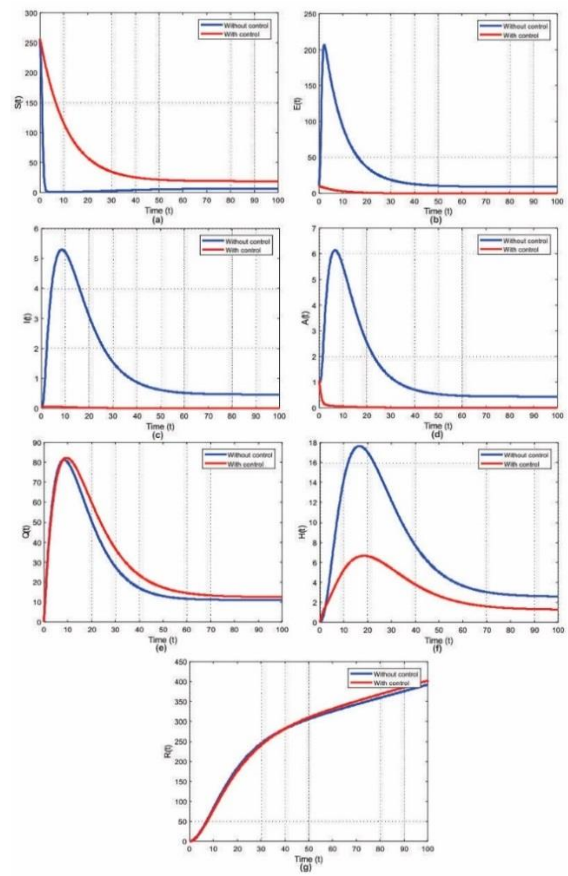

Fig 8: Optimal control simulation results with control u_1 and u_2

Furthermore, the most effective strategy is determined from the value of the objective function. Based on the simulation results, the objective function value of each strategy is as follows:

Table 2: Optimal Control Numerical Simulation

Strategy I	Strategy II	Strategy III
98.5659	2,843.7	90.1691

Based on comparing the objective function values in the table, the minimum objective function value is obtained by strategy III. Therefore, the most effective strategy is strategy III.

IV. CONCLUSIONS

The optimal control problem is solved by determining the objective or cost function, forming the Hamilton function to get the optimal system with the Pontryagin minimum principle. Optimal control simulations of the two a given strategy show that a combination of strategies (preventive action control through education and treatment of infected individuals) is the most effective strategy in minimizing the number of subpopulations (exposed, symptomatic and asymptomatic), as well as costs associated with control.

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